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Quantum Entanglement Oscillations

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Abstract Quantum entanglement is shown to exist as a means of lowering ground state energy for multi-component systems. Symmetric and anti-symmetric system wavefunctions are thus simply due to the inter-particle potential and not to fundamental particle types: fermions and bosons. The paper shows that additionally to the cases known, bosons—apart from the condensate minimum, can exhibit an energy minimum type allowing entanglement oscillations. This fundamentally new case could conceivably be the origin of the conflicting properties observed in super-solidity: lower (fluid-like) rotational inertia (Kim and Chan in Nature 427:225, 2004; J. Low Temp. Phys. 138:859, 2005), higher (solid-like) shear modulus (Chan in Science 319:29, 2008).

Keywords Quantum entanglement · BE-condensate

"In a letter to Alfred Landé (24 November 1924), Wolfgang Pauli announced an *extremely natural prescriptive rule* that could shed light on some puzzling spectroscopic phenomena he had dealt with in the past three years. The foundation of the rule remained an open question. Nevertheless, a few months later, in January 1925, Pauli announced it as if it were a commandment of nature: *In an atom there cannot be two or more equivalent electrons for which the values of all four quantum numbers coincide. If an electron exists in an atom for which all of these numbers have definite values, then this state is occupied.*" [4].

The full entanglement predicated by Pauli's Principle is relaxed however by theories such as fractional statistics [5] and anyon statistics [6], bringing back also the question of why should far apart quantum entities (considerably remote compared to their corresponding wavelengths) be correlated in any way. The present paper shows that proximity is essential

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to entanglement and that quantum entities entangle in reducing system-energy via interaction. This also shows that there is a time needed to "fall" into entanglement, respectively entanglement is an attribute of equilibrium states. It is noteworthy mentioning in this respect that entanglement has been observed more omnipresent than not and that suppressing [7] it is more challenging than its spontaneous occurrence.

Quantum entanglement has also been a source of conceptual difficulty in quantum mechanics—for the notion of "quantum indiscernability", which translated into practical terms basically denotes "identical and the same", give or take scientific expression. This notion makes a difference, un-explained one, between "classical" and "quantum" objects. To better understand the situation, we should position ourselves as receivers of some distant information, information that states the existence of 2 identical entities and nothing more. Unless more information is given, such as "the one at position A" and "the one at position B", or similar kinematic labels, the entities will never be discernable—classical, or quantum alike. The implications of conceptually understanding this situation are far reaching in avoiding physical discontinuity. Consider for instance two hydrogen molecules, one having in place of an electron a muon—a special muon, one with mass almost that of the electron. By Pauli's Principle, the wavefunction of the first is entangled, while the second one is not. As the mass of the "muon" equals that of the electron, the molecules become identical in all respects, other than an externally given label by us, while their energy levels still differ by the "exchange interaction" term. There is evidently no reason for such discontinuity solely on the basis that *we think* the particles differ.

It will be shown in this contribution that entanglement, although related to, pertains not as much to the abstract concept of "quantum identicality", as to the more physical and tangible requirement of energy minimum. This view will also reconcile the energetic discontinuity between systems containing identical, almost-identical and non-identical particles (as in the above example).

To show how energy minimum conditions entanglement, take a 2-particle system of Hamiltonian:

$$\mathcal{H} = H(1) + H(2) + V \tag{1}$$

Its wavefunctions $|\psi, \phi\rangle$ and $|\phi, \psi\rangle$, with $\|\psi\| = \|\phi\| = 1$, have the same energy. Interestingly, superpositions of these wavefunctions have either lower or higher energies due to the "exchange"-energy term, that are entanglement dependent:

$$\begin{aligned} |\text{status}\rangle &= \lambda|\psi, \phi\rangle + \mu|\psi, \phi\rangle \\ E &= \frac{\langle \text{status}|\mathbf{E}|\text{status}\rangle}{\langle \text{status}|\mathbf{1}|\text{status}\rangle} \end{aligned} \tag{2}$$

While the numerator of the above expression has the expected behavior, division by the denominator (to give "energy *density* per 100% of existence") gives a very interesting behavior. Firstly, the above can be better discussed function of the entanglement specific parameters (F, ζ):

$$E = E_{ent} + \Delta \frac{\upsilon + \zeta_0^2 - (\zeta - \zeta_0)^2}{1/F + \zeta^2} \tag{3}$$

where $|\phi\rangle = P_{\psi\phi}|\psi\rangle + |\chi\rangle$ with $\chi \perp \psi$, the projector $P_{\psi\phi} = \langle\psi|\phi\rangle$, $\Delta = E_{ent} - E_{uni}$ and $\zeta_0 = E_{exch}/2\Delta$; also:

$$
\begin{aligned}
\zeta &= \frac{|P_{\psi\phi}|}{\sqrt{1-|P_{\psi\phi}|^2}} \\
F &= \frac{|\lambda+\mu|^2}{|\lambda|^2+|\mu|^2} \\
v &= \frac{1}{\Delta} V_{\substack{\psi\,\chi\\ \chi\,\psi}} \\
E_{uni} &= 2H_{\psi\psi} + V_{\substack{\psi\,\psi\\ \psi\,\psi}} \\
E_{ent} &= H_{\psi\psi} + H_{\chi\chi} + V_{\substack{\psi\,\chi\\ \psi\,\chi}} - V_{\substack{\psi\,\chi\\ \chi\,\psi}} \\
E_{exch} &= H_{\psi\chi} + H_{\chi\psi} + V_{\substack{\psi\,\psi\\ \psi\,\chi}} + V_{\substack{\chi\,\psi\\ \psi\,\psi}}
\end{aligned}
\tag{4}
$$

Parameter F controls the degree of entanglement (0 for "fermionic" and 2 for "bosonic"), while ζ controls the overlap between ψ and ϕ (0 for no overlap, ∞ for total overlap). It can be seen that while for $F \to 0$ ("fermionic" entanglement) the energy does not depend on the ψ-ϕ overlap, for $F = 2$ ("bosonic" entanglement) there is a non-trivial ζ-dependence.

Energy minimum is *not* obtained by $d_{\zeta,F}E = 0$, as there are global aspects and the minimum is not always attained in a position of extremum—case $^{(-,+)(-,+)}$ in Fig. 1 for instance. Function of the signs of $(\zeta_0, v)\times(\Delta, \zeta_0^2 + v)$ Fig. 1 classifies all possible cases:

fermionic: $^{(-+)(-+)}$, $^{(--)(+-)}$, $^{(--)(--)}$, the wavefunctions in this case are anti-symmetric for energy minimum and the single-particle wavefunctions orthogonal,

bosonic: $^{(++)(+-)}$, $^{(++)(++)}$, $^{(+-)(+-)}$, $^{(-+)(-+)}$, the wavefunctions in this case are symmetric for energy minimum and the single-particle wavefunctions identical,

special: $^{(++)(-+)}$, $^{(++)(--)}$, $^{(+-)(--)}$, $^{(-+)(+-)}$, $^{(-+)(++)}$, the wavefunctions in this case are symmetric (bosonic-like) for energy minimum, however the minimum is for non-identical single-particle wavefunctions.

Note that the negative range of ζ (non-physical) was plotted only for mathematical continuity. It can be seen that function of H and V any given system reaches energy minimum in two states:

1. "fermionic"—$(F, \zeta) = (0, 0)$, or
2. "bosonic"—$(F, \zeta) = (2, \zeta)$, which for $\zeta = \infty$ is the BE-condensate.

There is thus nothing "intrinsic" to being a fermion or a boson, just the particle's interaction type and by consequence the associated wavefunction folding in reaching energy minimum. It should be noted that although it is usually spin that dictates the entanglement type of the energy minimum (hence if the particle is a fermion/boson), spin is just one component of the interaction, and the other terms may upset this categorisation—most likely in solids, where the lattice can modulate the effective interaction potential. Still, such hypothetical type "inversion", if it exists, should be expected under totally exceptional conditions.

Real-life is additionally complicated by the fact that participating particles may change the ψ and ϕ states with which they "choose" to participate, in search for the best partner states to entangle with and to form the energy minimum (within the bounds of angular/momentum and energy conservation in the radiative processes therewith associated).

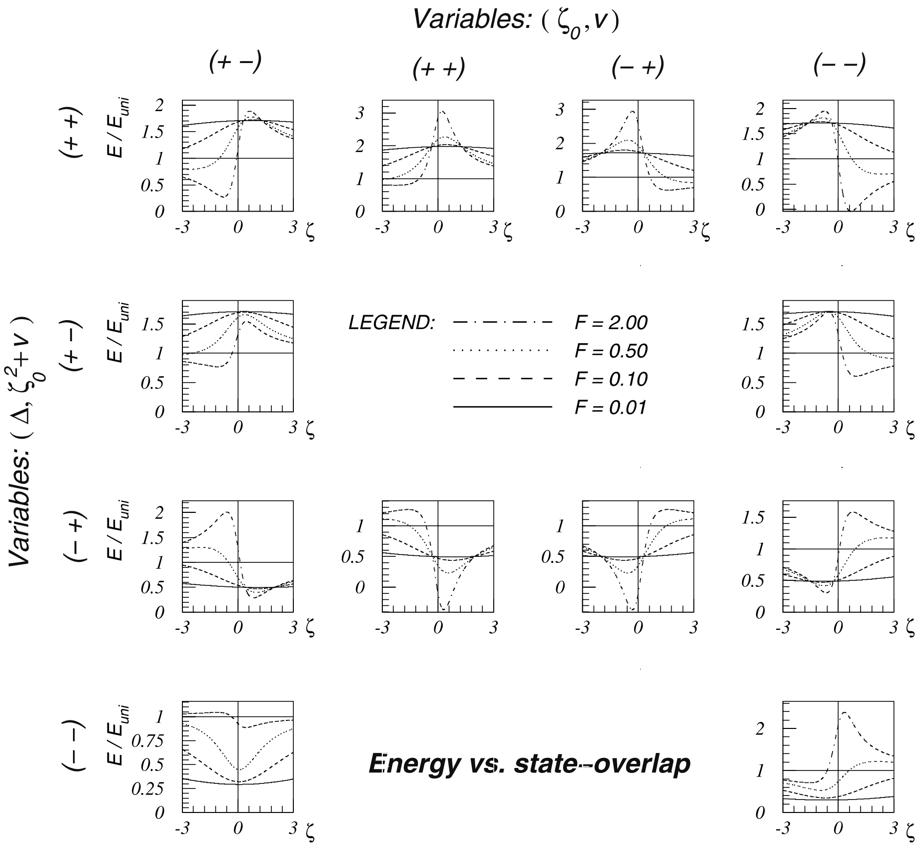

Fig. 1 Entanglement cases as energy versus the overlap parameter of the participant states (ζ). The *dash-dot curves* correspond to "bosonic" entanglement, $|\psi,\phi\rangle + |\phi,\psi\rangle$ and the *solid curves* to "fermionic" entanglement, $|\psi,\phi\rangle - |\phi,\psi\rangle$. An interesting new minimum is observed for bosonic-entanglement at $\zeta \neq \infty$ (non-identical wavefunctions) in certain parameter cases, that permits ζ-oscillations and could explain the strange properties of super-solidity (fluid vs. solid properties)

It can be seen here why the above case of forming an H_2 molecule, with one particle a "muon" of mass similar to the electron, entangles the two: the resulting state has least energy—regardless if the two particles are identical or not. Entanglement of the electron with the "muon" is imbalanced, but nonetheless existent: $\lambda|\psi_1,\phi_2\rangle + \mu|\phi_1,\psi_2\rangle$, where $\psi_1 \to \psi_2$ and $\phi_1 \to \phi_2$ as the "muon" (mass) converges to the electron. In this sense there is no discontinuity in the limit of the "muon" converging to the electron, and no conceptual discontinuity between identical, almost-identical and non-identical particles. Note that energy equation (2) also gives the answer to the puzzling question of what happens if two electrons participate with the same state ψ—ruled out by the Pauli Exclusion Principle. In this case the energy is $E = E_{uni}$, an energy usually quite high above minimum, hence not observed. Such system wavefunctions have not been studied, but may exist as non-equilibrium states and may be valuable study for ionisation, nuclear decay, etc processes.

The "special" case has not been documented before, and further emphasizes the fact that the concepts of fermions and bosons rather are effects, not basic building blocks in nature. In the special-case the energy minimum is always $F = 2$ (symmetric, bosonic) entanglement, however at $\zeta \neq \infty$ (non-condensate). This interestingly allows for ζ-oscillations between different overlap degrees of the participating states—in the cases shown in Fig. 1 on the order of $|P_{\psi\phi}|^2 = 20\dots100\%$, with the associated energy oscillations on the order of $10\dots50\%\ E_{uni}$. Since the special case is obtained by parameter "tuning", it is conceivable that this is also what happens in the case of ^{4}He cooling, the lattice modulating the integration potential and tuning the parameters such as to cross through regions of "special" entanglement "oscillations"—between $\zeta = \infty$ and $\zeta \neq \infty$. Conceivably this could be the source of the conflicting properties observed in super-solidity: lower (fluid-like, condensate, $\zeta = \infty$) rotational inertia [1, 2], with higher (solid-like, $\zeta =$ finite) shear modulus [3].

Another interesting feature of energy equation (2) is that:

$$E_{|\psi,\phi\rangle} = E_{|\phi,\psi\rangle} = E_{|\psi,\phi\rangle \pm i|\phi,\psi\rangle} \tag{5}$$

In fact for any $F(\lambda,\mu)$ there are an infinite number of combinations $\lambda'/\lambda = \mu'/\mu$ such that $E_{\lambda'\mu'} = E_{\lambda\mu}$—allowing the system to migrate without energy effort to other (more/less entangled) *palier*-states. This perhaps could explain some of the low energy nuclear phenomena [8–11]. Bound multi-particle systems are formed by releasing the extra "exchange"-energy formed through entanglement. It is thus logical that disentangling the system will require this energy back. However, entanglement changes also spins, angular momentum, making it thus non-trivial to disentangle a system due to the conservation laws applicable. All systems possess "palier"-states, though some are at $\zeta < 0$ (non-physical), or at $\zeta = \infty$ (BE-condensate)—Fig. 1. The rest have one, or two (in the special-case) palier-states. Rising the system to a palier-state via some energy kick allows thereafter system disentanglement without energy effort by an external probe interacting with *only one* (disentangled) constituent.

For multi-particle ($N > 2$) systems there are many more foldings possible. Fermionic systems can couple for instance in antisymmetric pairs, with these pairs further coupling as a BE-condensate—as in the theory of superconductivity [12]. The notion of part-entanglement thus arises, together with that of the meta-entanglement of such sub-sets. It is debatable if the absolute minimum is the totally entangled state, or just a local one. The proof below shows that the totally entangled state is a at least a local minimum (possibly the absolute one). For the N-particle system, the state can be formed as a sum over all permutations π:

$$\begin{aligned} |\text{status}\rangle &= \sum_{\pi} \lambda_\pi |\psi_{\pi(1)}, \psi_{\pi(2)}, \dots, \psi_{\pi(N)}\rangle \\ \mathcal{H} &= \sum_{i=1}^{N} H_{(i)} + \sum_{i>j}^{N} V_{(ij)} \\ E &= \frac{\langle \text{status}|\mathbf{E}|\text{status}\rangle}{\langle \text{status}|\mathbf{1}|\text{status}\rangle} \end{aligned} \tag{6}$$

The above expression differentiated, $d_{\lambda_\pi} = 0\ (\forall)\pi$, will give local energy minima:

$$\mathcal{H}_{\pi(\Psi),\Psi}\mathbf{1}_{\Psi,\Psi} = \mathcal{H}_{\Psi,\Psi}\mathbf{1}_{\pi(\Psi),\Psi} \quad (\forall)\pi \tag{7}$$

which is an eigen-value equation, with $|\Psi\rangle = |\text{status}\rangle$. In expanded form this is:

$$\sum_{q,s,t}^{N} \lambda_{(\pi q)}\bar{\lambda}_s\lambda_{(st)}(\mathcal{H}_{1,q}\mathbf{1}_{1,t} - \mathcal{H}_{1,t}\mathbf{1}_{1,q}) = 0 \quad (\forall)\pi \tag{8}$$

with π, q, s, t permutations. The above is realised for:

$$\sum_{q,s,t}^{N} \lambda_{(\pi q)} \bar{\lambda}_s \lambda_{(st)} = \sum_{q,s,t}^{N} \lambda_{(\pi t)} \bar{\lambda}_s \lambda_{(sq)} \quad (\forall)\pi \tag{9}$$

On physical grounds, the system's energy must be invariant to particle permutations, thus:

$$\lambda_\pi = f(\mathrm{sgn}_\pi) = a + b \cdot \mathrm{sgn}_\pi \tag{10}$$

hence $|ab| \cdot \mathrm{sum} = 0$, where the non-zero sum term is:

$$\sum_{q,t}^{N} \left(e^{i\phi_a} |b| \mathrm{sgn}_t + e^{i\phi_b} |a| \mathrm{sgn}_{\pi q} \right) (\mathcal{H}_{1,q} \mathbf{1}_{1,t} - \mathcal{H}_{1,t} \mathbf{1}_{1,q}) \tag{11}$$

Therefore $ab = 0$, respectively $\lambda_\pi = \mathrm{const} \cdot (\mathrm{sgn}_\pi)^k$, $k = 0, 1$—as already known (totally anti/symmetric wavefunctions, depending on the interaction).

The present contribution has shown that the archetypal categories of fermions and bosons correspond to the interaction type and the inherent folding (entanglement) of the system wavefunction in the quest for energy minimum. It is also shown that a new type of bosonic behavior (non-condensate), that allows entanglement oscillations, is to be expected in novel areas of parameter space.

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